

Exam

Spring 2020

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Amalie, Bente and Cecilie live together during Corona lockdown. They know they will need to spend much time at home, so they want to make their balcony nice. If two of them contribute it will be a nice balcony. If only one of them contributes, it will look sad. There are four possible outcomes from each of their perspectives:

1. The other two contribute and she doesn't: Payoff = 4
2. She and one or both of the others contribute: Payoff = 3
3. She doesn't contribute and only one or neither of the others: Payoff = 2
4. She is the only one who contributes: Payoff = 1

(a) Write up this game in a bimatrix with 3 players (hint: Amalie chooses the matrix). Find the Pure Strategy Nash Equilibria.

Solution: Contribute = C and Don't Contribute = D C,D,C and C,C,D and D,C,C and D,D,D

(b) Draw this game as a dynamic game. Amalie goes first, then Bente, then Cecilie. It is not necessary to upload the drawing, but you might need to draw it for yourself to answer the following questions.

(c) How many strategies does each of the players have in the dynamic game?

Solution: Amalie has 2, Bente has 4 (2x2) and Cecilie has 16 (2x2x2x2).

(d) What is the SPNE outcome of the dynamic game? What actions will each of the girls take on the equilibrium path of the dynamic game?

Solution: Since Amalie goes first she can defect, knowing that the other two will then cooperate. You need to specify the actions the girls would take off the equilibrium path. SPNE: (D, DC, DCCD). In this version Amalie will play D and then Bente and Cecilie will both play C.

(e) Why do we need to specify which moves someone would take "off the equilibrium" path?

Solution: First, we need to know what would happen, if the player would end up at a node by mistake. Second, we need to know what will happen off the equilibrium path, to know how people will behave on the equilibrium path (i.e. whether they will have an incentive to deviate).

- (f) Consider your solution for d). What might change if we would model this as a psychological game and introduced reciprocity as a motivating factor for choosing an action. Do you think the outcome would be the same? Why? Why not? Explain in 2-3 sentences.

Solution: In d) Amalie defects knowing that the other two will decorate their balcony. If we include reciprocity as a motivation then the other two girls might want to punish Amalie for not contributing and also not contribute, even though they would be better off based on the payoffs we defined in the classic game.

2. Danish society is playing the following stage game G during corona. We simplify the situation into a two player game. The actions are "S" (staying home) vs. "G" (going out). It is best to go out alone because you cannot get sick and everyone staying home is better than everyone going out. The game is repeated indefinitely. Every day is one period of the game. Everyone is impatient to get back out, so their discount factor is β with $0 < \beta < 1$. The following strategies are available for the two players:

- "Go Out" = Always go out
- "Stay in" = Always stay in
- "Trigger" = Stay in until the other one goes out and then always go out
- "Tit for Tat" = Start by staying in. After that play what the other one played in the previous round.

The Stage Game G:

		Society	
		S	G
You	S	2, 2	-1, 3
	G	3, -1	0, 0

- (a) Calculate the payoffs for the strategy profiles (Tit for tat, Tit for tat) and (Trigger, Trigger)

Solution: Both strategy profiles result in both players playing "stay in" in each repetition. That leads to payoffs being: $2 + 2\beta + 2\beta^2 + \dots = 2/(1 - \beta)$

- (b) Show that neither (Go out, tit for tat) nor (Go out, Trigger) are SPNE. Explain briefly.

Solution: In both cases "Go out" is the best response to "tit for tat" or to "trigger". So they cannot be SPNE. Best response against "go out" is "go out".

- (c) Show that (Go out, go out) is a SPNE. Explain briefly.

Solution: (G, G) is a pure strategy nash equilibrium in the stage game. Thus (Go out, go out) is also a SPNE in the repeated game.

- (d) Show that (Stay in, stay in) is not a SPNE. Explain briefly.

Solution: "Go out" is the best response to "stay in" than "stay in", so (Stay in, stay in) is not a SPNE.

- (e) For which value of β is (Trigger, Trigger) a SPNE?

Solution: We calculate the one round deviation payoff and compare it to the payoff from not deviating:

$$3 + 0 * \beta + 0 * \beta^2 \dots = 3 \leq 2/(1 - \beta)$$
$$\beta \geq 1/3.$$

- (f) If the lockdown was announced to last exactly 14 days, what would be the unique SPNE of the game? Write a short proof for your answer.

Solution: If the lockdown would only last 14 days, then it is a finitely repeated game. If G has a unique Nash Equilibrium, then $G(14)$ has a unique subgame perfect Nash Equilibrium in which the Nash Equilibrium of G is always played. The NE of the stage game is (G,G). The SPNE is (Go out, Go out). Proof: In the last period, the players will play the unique NE (because of subgame perfection). Then in the second-to-last round, the players cannot influence what happens in the last round, and will also play the NE! And in the round before that, and so on.

3. In the height of the pandemic there were several bottles of hand sanitizer up for auction on auction sites such as eBay.

- (a) What is the benefit for the seller to sell the sanitizer in an auction vs. selling it with a fixed price? Explain briefly.

Solution: It is difficult to know what the valuation of the potential buyers are in this situation. It is a new situation and some people might have a very high valuation for the good. The good goes to the bidder who values it most.

- (b) Hand sanitizer can be argued to have "private value" or to have "common value". Define what these terms mean and then give one argument for why hand sanitizer should be treated as a private value good and one for why it should be treated as a common value good.

Solution: Private values are independently distributed values for a good while common values apply when goods have a clear resell or market value and every bidder values them the same. In the hand sanitizer case it can be treated as a private value because people have different risk profiles and different jobs, so they have different use for hand sanitizer. It can be treated as a common value because it could likely be resold at market price.

- (c) eBay uses an ascending first price auction format with purely private values. Explain what a bidder's best strategy would be in an ascending first price auction. How should the bidder determine his maximum bid and when should he bid? Why? 4-5 sentences.

Solution: The profit from a first price auction with private values comes from the price paid minus the valuation. This should be maximized. Bidding aggressively gives higher chance of winning the auction, but lower payoff conditional on winning. So unless there is some bid shading, the bidder cannot make any profit. There is only a positive payoff if the bidder wins the auction. The more people are in the

auction, the less he can shade his bid and make a profit. It is best to bid as late as possible so the other bidders do not have a chance to overbid - this is also called sniping.

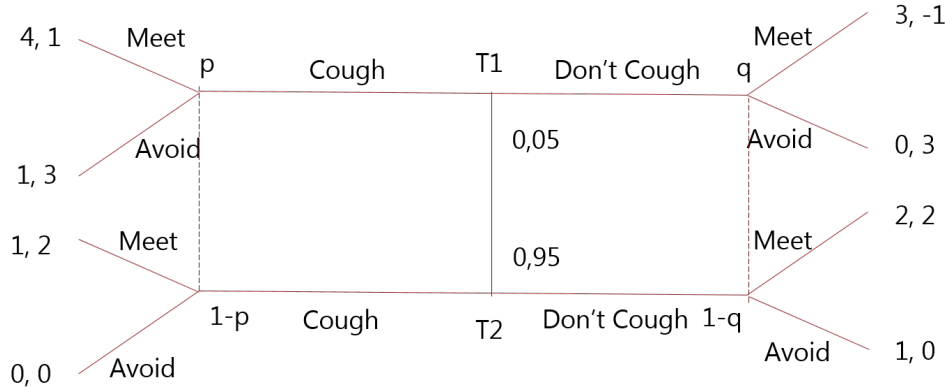
- (d) How would the strategy be different in an ascending second price auction with purely private values? What should the maximum bid be and when should you bid? Why? 4-5 sentences.

Solution: In a second price auction, the winning bidder pays the price of the highest loser's bid. The optimal strategy is to bid his own valuation. If he bids more, he will still win and still pay the second highest bid, if he bids less there is a risk of losing the auction even though he could have been at least as good off when bidding his valuation. Again it makes sense to bid at the last possible moment so no information about his valuation is transmitted to the other bidders.

- (e) What problem can occur in a sealed-bid first price auction with common values that is not a problem in a sealed-bid first price auction with private values? Why does this happen? 2-3 sentences.

Solution: In a common value auction there is a risk of the winner's curse. The winner's curse arises when a bidder fails to take account of the fact that, when he wins, he is likely to have made an overly optimistic estimate of the object's value. If bidders receive signals about the value of a good, there is a chance that some will receive very high signals. If they act on them, they will have paid too much.

4. Consider the following game G'. There are two types of senders. T1 has tested positive for Covid-19, T2 is healthy. The receiver has to decide whether to meet up with their sender friend after they talk on the phone. They either hear the friend coughing or not. The receiver is sad if they miss out on meeting their friend, but they are very happy when they avoid getting sick from Covid. The sender with Covid (T1) is very lonely and wants to meet their friend. Possible messages are: C (Cough) and D (Don't Cough). Possible actions for the receiver are: Avoid (a) and Meet (m).



- (a) Is G' a game of complete or incomplete information?

Solution: By definition, G is a game of incomplete information. The receiver does not know which type the sender is.

- (b) Find a pooling equilibrium where both sender types play D. Show the steps of how to get to the solution. Explain your process.

Solution: Pooling equilibrium: (DD, am, $q = 0.05$, $p \geq 1/2$).

- (c) Show that there are no separating equilibria. Which type wants to deviate?

Solution: Separating equilibrium: Not possible. Type 1 always wants to deviate.

- (d) There is a sick and a healthy sender type. Explain intuitively, by interpreting the payoffs, why there are no separating equilibria in the case shown in the game. Explain intuitively (without showing it formally) how the payoffs of the sender would need to change in order for both sender types to prefer a separating strategy.

Solution: Both sender types get the highest payoff from seeing their friend, but the friend only wants to meet with the healthy type. So the sick type will try to pool with the healthy type. If for example, the sick type would want to protect their friend and get a higher payoff from not meeting them when sick, then they would prefer to play a separating strategy where the sick one coughs and the healthy one doesn't cough.

- (e) Now imagine a situation where the sender says "I am not sick". This message is costless to send. What do we call these types of games?

Solution: This is a cheap talk game.

- (f) What are the three conditions that need to be fulfilled for this message to be effective? Explain what all three conditions mean in this Covid example.

Solution:

1. Sender types prefer different actions: Healthy type want to meet, sick type does not want to get her friend sick.
2. Receiver types prefer different messages from different types: Prefers that healthy type says she is not sick and sick type says that she is sick.
3. Sender and receiver incentives are somewhat aligned: Both want to meet if healthy and not meet if not healthy.

5. In the UK at the beginning of the lockdown, there were some food shortages in the supermarkets. The supermarkets decided to design a policy that would allow older citizens to shop first. For the first hour of the day, only individuals over 65 were allowed to shop. What unintended consequence do you think could result from this policy. Suggest one possible fix to the problem you have mentioned.

Solution: Possible answers: Crowds of seniors and other vulnerable groups now all show up at the same time. The time is more limited than the usual shopping hours. Younger people might ask older citizens to buy things for them when they are available. Solutions: Limiting the amount per person/household. Shelves only for elderly or service

by request. Sending groceries home instead of asking elderly to come shop. Special websites for elderly. Other answers possible, if in line with problem detected.